Advanced Algorithms — Exercise Set 2

Name: ____________________________

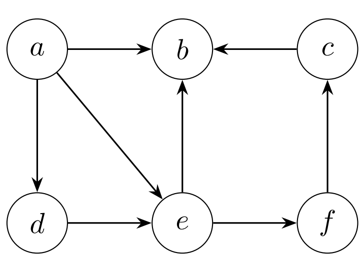
- Submit in class on **February 10, 2026**.
- Feel free to discuss with others, but write up your own solutions.
- This will be graded half for completion and half for correctness.
- Remember that there is an assignment due on Feb 17, this exercise set is shorter than usual so that you have time to work on the assignment.

Network Resiliency

Let's say you're in charge of a mobile phone company. You set up a network of cell phone lines across the country so that many towns and cities are covered. Of course, you also want to ensure a consistent and reliable experience for all your customers. Since cell-phone lines commonly go down due to any number of reasons, you want to make sure to build some redundancy into your network. This is called **fault-tolerant network design**, and is useful for all sorts of applications, including transportation networks, energy networks and even supply chains.

The redundancy of a network to edge failures is formalized by the following definition. We use a directed graph for concreteness but a similar definition can be made for undirected graphs too. We call a set of paths *edge-disjoint* if they do not have any edges in common.

Definition. *In a directed graph G, we say that two vertices a and b are **2-edge-connected** if there are two edge-disjoint paths from a to b in G. More generally, a and b are k**-edge-connected** if there are k edge-disjoint paths from a to b in G.*



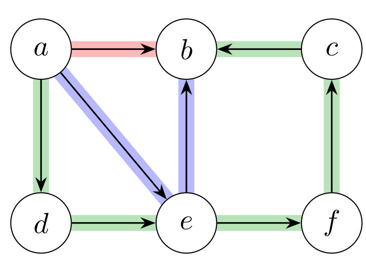


Figure 1: Vertices a and b are 3-edge-connected because there are three paths from a to b which share no edges: $\{a \to b\}$, $\{a \to d \to e \to f \to c \to b\}$, and $\{a \to e \to b\}$.

Problem 1. *What is the edge-connectivity from a to f in the graph pictured above?*

Solution.

Problem 2. *The reason that k-edge-connectivity is often used as a measure of fault tolerance is that it encodes how many edges can fail while preserving connectivity. Explain why, if two vertices a and b are k-edge-connected in a graph G, then there will still be a path from a to b even after any set of $k-1$ edges are deleted from G.*

Solution.

Problem 3. *The notion of network resiliency is very related to flows and cuts. Can you see how to compute the edge-connectivity between a pair of vertices using flows? Given a directed graph G, and two vertices s and t, give an algorithm to compute the maximum edge-connectivity between s and t. (Hint: what should be the capacity on each edge?).*

Solution.

Problem 4. *Explain why, if s and t are k-edge-connected in G, then the minimum $s-t$ cut value is at least k.*

Solution.

Problem 5. *Show that k-edge-connectivity is **transitive**. That is, if there are k edge-disjoint paths from a and b, and k edge-disjoint paths from b and c, then there are k edge-disjoint paths from a to c. [Hint: use the observation in problem 4]*

Solution.